Capacity Reserves in the Global Buckling Analysis of Steel Columns

Peter KNOEDEL
Professor
Augsburg University of Applied Sciences, Germany
info@peterknoedel.de

Alfred MUELLER
Senior Engineer
hmb engineering
Karlsruhe, Germany
info@hmb-ingenieure.de

Markus HAFNER
Senior Engineer
hmb engineering
Memmingen, Germany
info@hmb-ingenieure.de

Asmaa ABUL OLA
Structural Engineer
hmb engineering
Karlsruhe, Germany
info@hmb-ingenieure.eu

Curriculum vitae

Peter Knoedel IWE, chartered engineer, obtained his PhD in civil engineering at the University of Karlsruhe. He runs a consultants office, concerned with fatigue and seismic design. He is Professor of Steel Structures at the University of Applied Sciences, Augsburg.
Alfred Mueller IWE, chartered engineer, obtained his diploma in civil engineering at the University of Applied Sciences in Karlsruhe, where he is visiting lecturer in steel structures. He runs a consultants office, mainly concerned with structural engineering and fire prevention.
Markus Hafner, chartered engineer, obtained his diploma in civil engineering at the University of Applied Sciences in Biberach. He runs a consultants office where he is mainly concerned with structural engineering and fire prevention.
Asmaa Abul Ola obtained her diploma in civil engineering at the University of Applied Sciences in Karlsruhe. She works as a structural engineer. One of her main fields of interest is energy efficient design.

Summary

In steel skeleton multi-storey buildings the global buckling analysis of columns is very often carried out on basis of individual single span members regarded as cut out of the system. With continuous columns and similar heights between the floors the section in the bottom storey will be governing due to the highest compressive load. In order to be on the safe side and to receive quick results it is common practice to have this section analysed with pinned ends.
Capacity reserves result from the fact that both, the stepped compressive load and the actual restraints at the column base, shorten the buckling length of the column and thus increase the buckling load. These effects are investigated in the present paper for practical column dimensions. It is shown, in which cases it is worthwhile to undertake a higher effort in structural analysis.

Keywords: *steel columns, buckling restraints, imperfections, buckling length, elastic subsoil, second order theory, base-plate, clamped support, stepped normal load, bottom support, multi storey column, buckling load,*

1. Introduction

In steel skeleton multi storey buildings such as car parks or office buildings the global buckling analysis of columns is very often carried out on basis of individual single span members regarded as cut out of the system. Appropriate rules for this method are given in EC3 [1] [2]. With continuous columns and similar heights between the floors the section in the bottom storey will be governing (*see Fig.1*). It is common practice to have this section analysed with pinned ends in order to be on the safe side and to receive quick results [3].

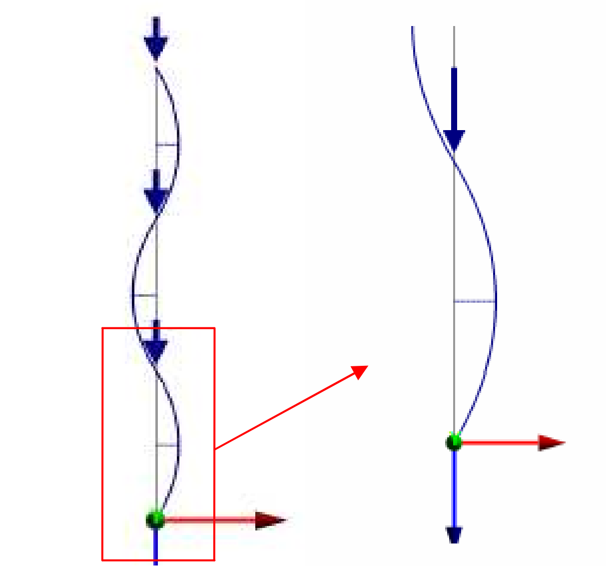

Fig.1: Example of a multi storey column with load and imperfection

In the calculation of the bottom storey column with pinned ends, the imperfections and the deflections in all storeys based on second order theory are assumed to be equal.
Due to the imperfections every section of the column is subjected to a bending moment.
As the normal forces of the upper column sections are lower than the normal force of the bottom section, also their bending moments are lower than the bending moment of the bottom section of the column. Therefore the deflections, we get from the calculation second order theory, are lower than those of the bottom section. The characteristics of the deflections within the multi storey column change.

For the calculation of the bottom section an elastic end-restraint at the upper support develops, which is neglected in the calculation with pinned ends. In the second paragraph, we analyse the influence of this effect on the buckling length of the bottom section.

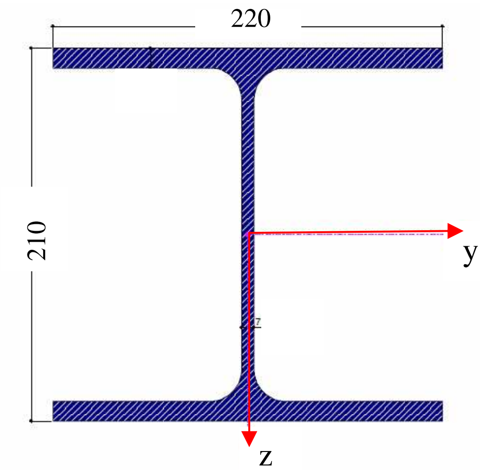

Fig.2: Example of a cross section with y- and z-axis

The second feature, that affects the buckling of the bottom section of the column, is that we do not have a real pin as support on the bottom of the column. Our structural member always has a cross section with real dimensions in its local y- and z-direction (*see Fig.2*) and a base-plate as connection to the foundation. For buckling the column has to tilt over its own edge on its bottom support. Caused by the normal force acting in the centre line of the column, a bending moment on the bottom support of the column develops, which operates in opposition to the buckling direction.

Additional there is a tension rod connection between base-plate and foundation at hinged connections, usually located in the middle of the cross section next to the web of the column, which supports the effect of the bending moment.
In the third paragraph, we look into this effect on the buckling strength, regarding also the adverse effect of an elastic foundation.
The third effect, which is not analysed in this paper but will be investigated in a future study, is the influence of the eccentric load on the buckling length of the bottom section.

2. Elastic restraint at the intermediate support

As already explained, in a multi storey building with approximately equal span lengths equal storey heights and equal loads the normal forces increase by equal increments in every storey from the top to the bottom. Therefore the bending moments and maximum deflections calculated with assumed imperfections based on second order theory also increase from top to bottom.

Within the imperfection shape the reversal point is located on the upper support of the decisive bottom section (*see Fig.1*). Caused by the above mentioned characteristics of the internal forces the reversal point of the deflection curve moves down towards to the bottom support of the column. An elastic restraint develops at the intermediate support of the column and reduces the risk of buckling and the buckling length (*see Fig.3*).

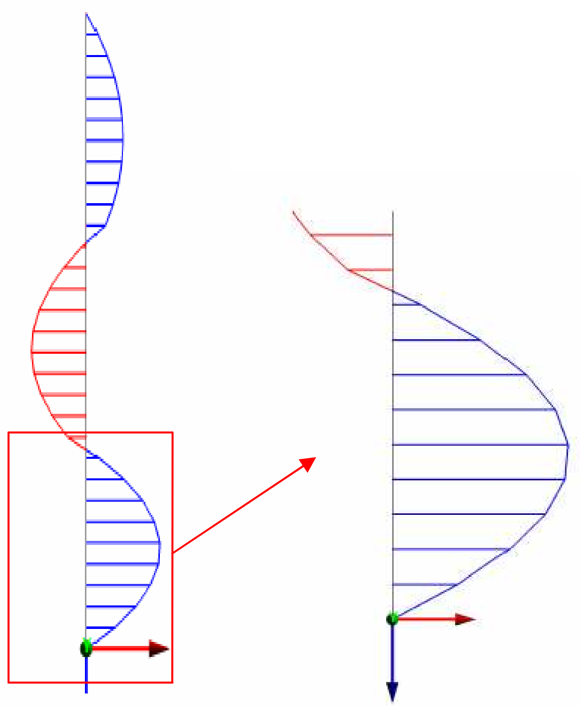

Fig.3: Elastic restraint at the upper support of the bottom section of the column

The influence of this elastic restraint has been analysed in a parametric study.

To get results for practical applications and a presentable range of results within these studies the storey height varies between 3m and 7m, the number of storeys varies between 2 and 8, the cross sections varies HE-200A, I_y= 3690 cm^4 and HE-400A, I_y= 45070 cm^4, the storey force varies from 100 kN to 900 kN and the imperfections vary from L//200 to L/1000.

We consider an example with 4m storey height, 3 storeys, cross section HE-220A, 300 kN force per storey and an imperfection of L/300. These parameters are varied. Only the cross sections vary on the base of 4 m storey height, 5 storeys, 300 kN force per storey and an imperfection of L/300 to show that the results do not change, even if more than one parameter is varied.

For every variation of the parameters the restraining bending moment on the intermediate support of the column is related to the maximum bending moment within

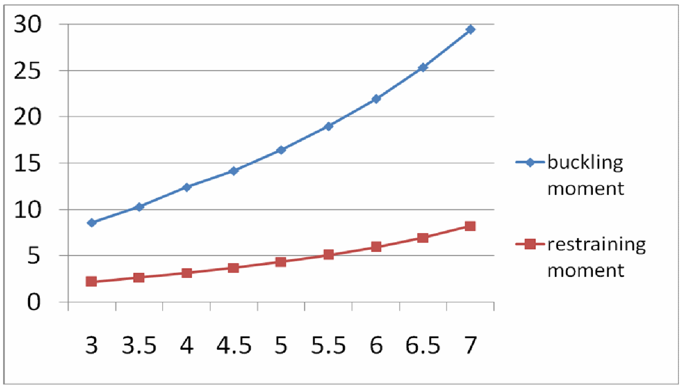

Fig.4: Bending moment and restraint moment varied by changing storey height

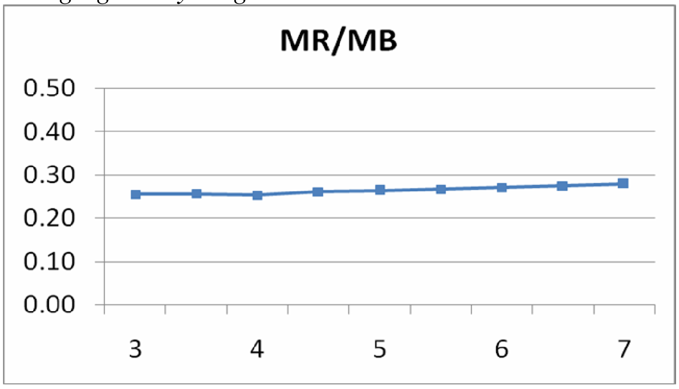

Fig.5: Restraining moment compared with bending moment varied by changing storey height

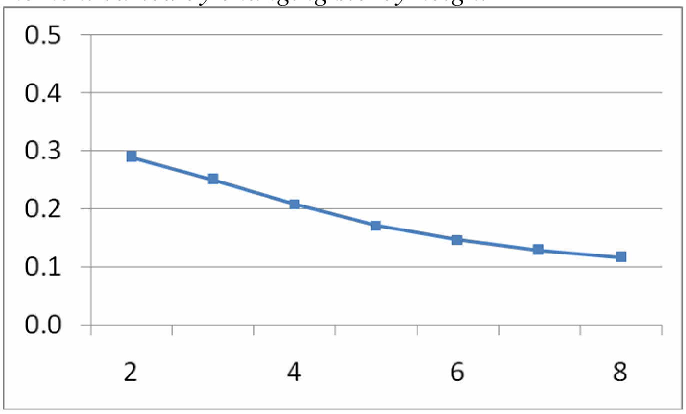

Fig.6: Restraining moment compared with bending moment varied by changing numbers of storeys

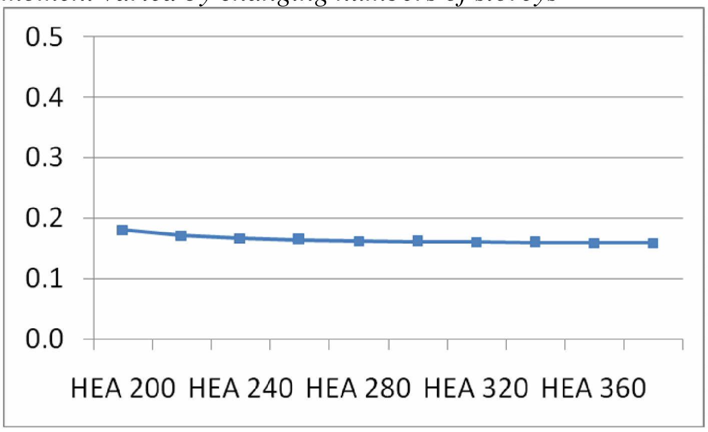

Fig.7: Restraining moment compared with bending moment varied by changing cross sections

the buckling length to get an idea of the influence of the elastic restraint on the buckling length and how it varies due to changing parameters.

As in *Fig.4* shown both the buckling moment and the restraining moment grow with increasing storey height as expected.

In *Fig.5* we can see that the ratio of restraining moment and buckling moment varies only between 0,25 and 0,28. That means that a ratio of 0,25 can be taken on the safe side for any storey height.

Fig.6 points out that the ratio changes significantly with the variation of the number of storeys. This aspect is not surprising when we look at the ratio of the normal forces of the bottom section and the next column section above. In a two-storey-building the normal force ratio of the upper column section and the bottom section is 0,5. That leads to a restraining moment/bending moment ratio of 0,3.

In a 6-storey-building the ratio of the normal force of the upper column section and the bottom section is 0,83. Also the difference of bending moment and deflections of both column sections is only 17 %.

Therefore the elastic restraining effect on the intermediate support of the column decreases to 0,15 for a 6-storey-building and 0,11 for an 8-storey-building, but still it influences the buckling behaviour.

On the contrary to the variation of the number of storeys the variation of the other parameters does not influence the ratio between restraint moment and buckling moment significantly.

Fig.7 shows that there is almost no variation of the ratio due to changing cross sections.

With increasing normal forces per storey again the ratio increases from 0,25 to 0,28, which is negligible, as we can see in *Fig.8*.

There is no apparent difference in the ratio depending on the variation of the imperfections as it is shown in *Fig.9*.

With these results a ratio of the restraining moment and the buckling

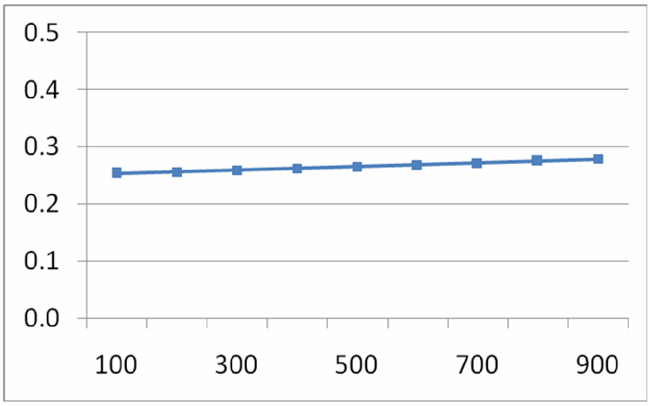

Fig.8: Restraining moment compared with bending moment varied by changing normal forces per storey

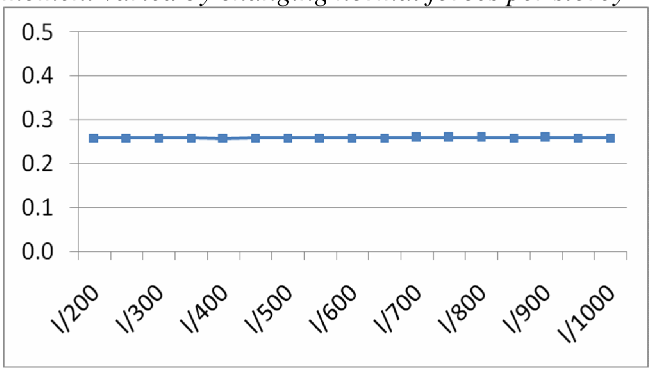

Fig.9: Restraining moment compared with bending moment varied by changing imperfections

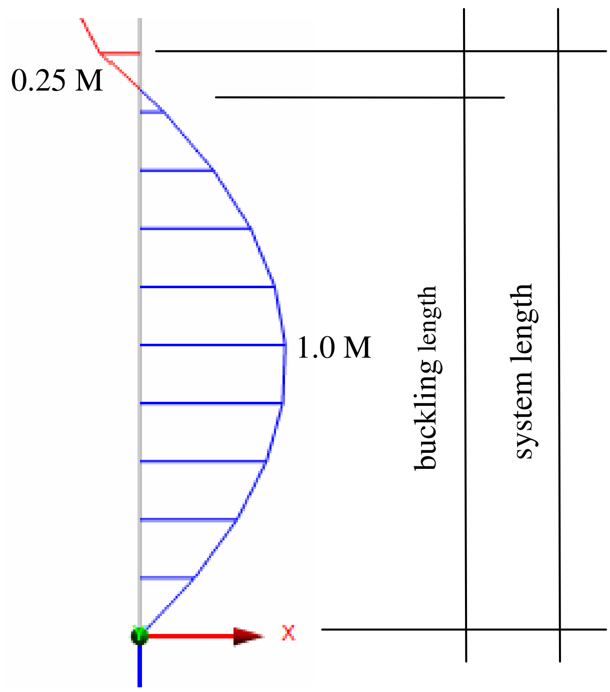

Fig.10: Reduction of the buckling length depending on the upper restraint effect

moment for a two-storey-building can be assumed with 0,25 on the safe side and with every additional storey the ratio decreases by 0,03 almost according to a linear function.

With these results, we know that there is a restraint effect at the intermediate support of 25 % down to 10 % depending on the increasing number of storeys. This restraint effect causes a change of sign and a zero crossing within the bending moment curve (*see Fig.3*).

From engineering mechanics we know, that at the zero point of the bending moment curve, there is a local maximum of the curvature for the bending moment is the derivation of the curvature.

Further the curvature is the derivation of the deflection. That causes a reversal point in the deflection shape at the local maximum of the curvature respectively at the zero crossing of the bending moment curve.

Knowing that a fully clamped support at one end of a regarded column section reduces the buckling length of the column section to 0,7 of its system length and that 0,7 of the system length of a column section with one hinged end and one clamped end is the length between the reversal points of the deflection shape respectively the length between the zero crossings of the bending moment curve, we can point out that there is reduction of the buckling length due to the restraint effect of the stepped normal forces within a multi storey column (*see Fig.10*). The reduction of the buckling length is depending on the zero crossing of the bending moment curve. With the knowledge of the ratio of restraint moment to buckling moment depending on the number of storeys which is worked out above the zero crossing of the bending moment curve can be calculated by means of engineering mechanics [4] [5] [6] and the buckling length of the decisive bottom section of the column can be reduced due to the upper restraint effect.

3. Elastic restraining effect at the bottom end of the column

At the bottom end of the column there is the connection to the foundation, usually by base-plate. Instead of a pinned end, as it is assumed in the conservative calculation, the column has real dimensions in x-and y-direction there (*see Fig.11*).

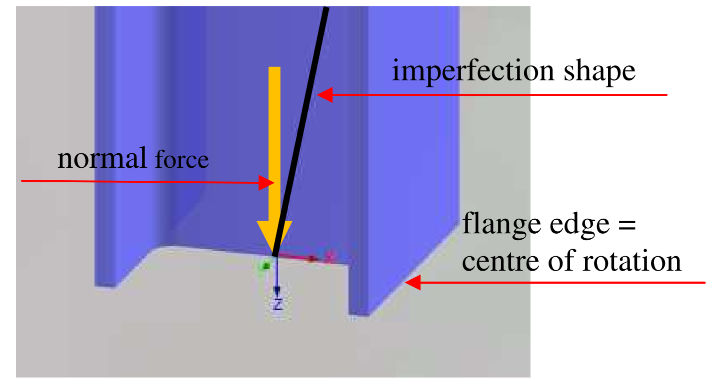

Fig.11: lower column end, showing the imperfection curve and the normal force

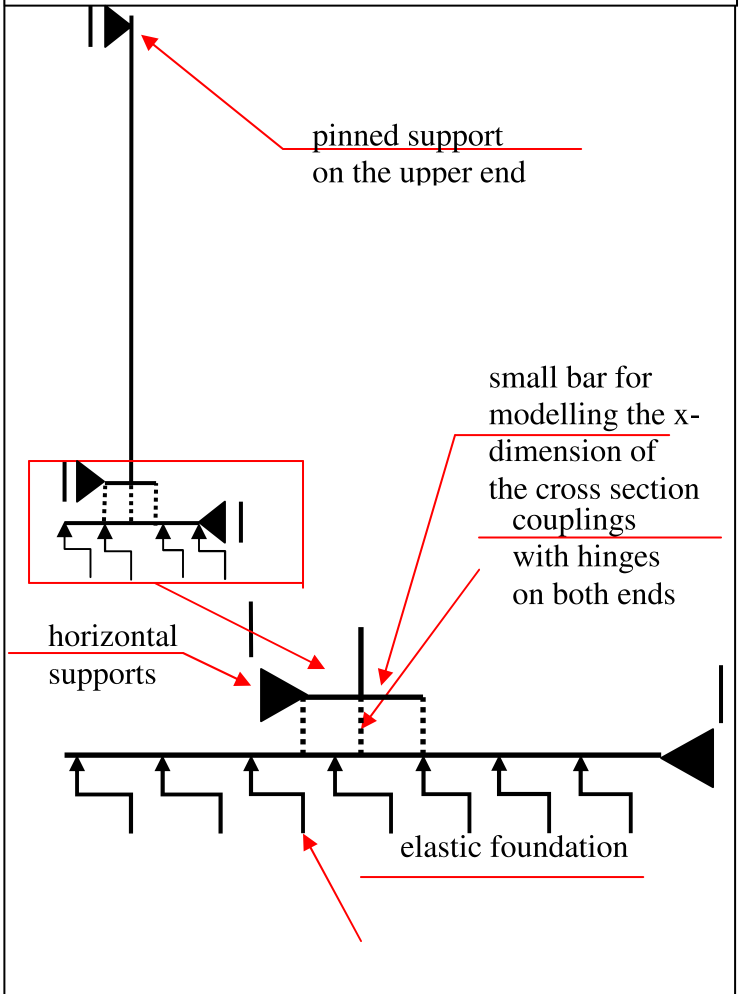

Fig.12: modelling of the elastic restraint at the bottom support

When buckling into the positive x-direction, as the imperfection shape shows, the column has to tilt over the marked edge (*see Fig.11*), which then is the centre of rotation. The normal force, acting in the centre line of the cross section, presses the bottom cross section back to his initial position, always rotating against the buckling direction and against the buckling moment. Therefore at this end a restraint for the column develops, which is very rigid, looking only at the connection between column and foundation. Regarding the elasticity of the subsoil, this restraint becomes more elastic. On a very rigid subsoil, the rotation of the foundation is very small and the restraint effect on the column is very rigid. On the contrary the rotations are bigger for a foundation standing on a weaker subsoil, the restraining moment decreases and therefore the influence of this elastic restraint on the buckling of the column decreases.

For studying the influence of this elastic restraint and the influence of the modulus of the subsoil, the following model has been chosen (*see Fig.12*):

For this analysis of the separate elastic restraint of the bottom support, the upper support is assumed as pinned.

On the lower end of the column the x-dimension of the column is modelled by attaching a small bar with a rigid connection perpendicular to the column. This bar is connected to a second bar by three couplings with hinges on both ends.

This second 1m long elastic foundation bar consists of a 1m/1m-concrete cross section. To ensure the numerical stability of the system, the necessary horizontal pinned supports have been modelled (*see Fig.12*).

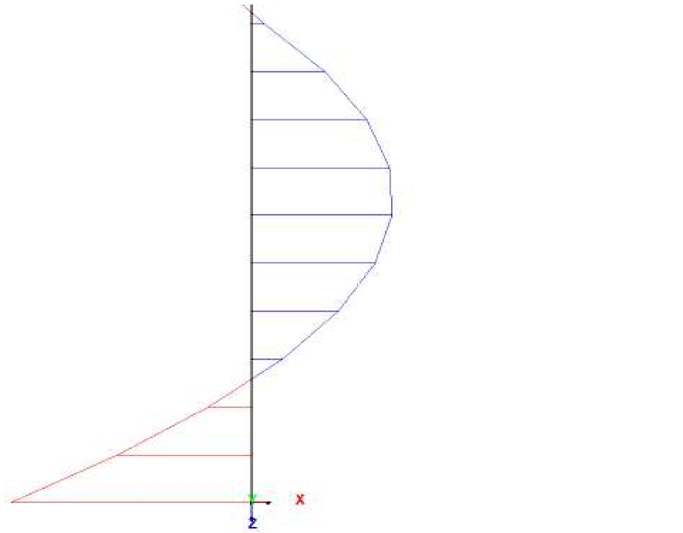

Fig.13: Example for restraint moment at bottom support for very rigid subsoil

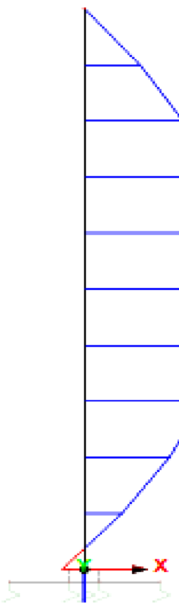

Fig.14: Example for restraining moment at bottom support for subsoil with very low modulus

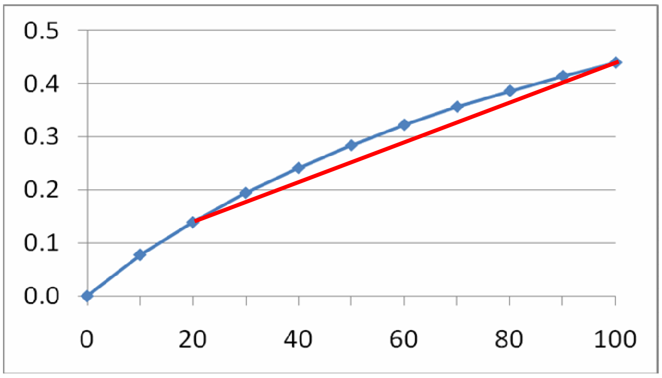

Fig.15: Ratio of restraining moment and bending moment depending on the modulus of the subsoil

The bending moments shown in *Fig.13* result from a calculation of a column with HE-220A as cross section, 300 kN as normal force, an imperfection of L/300 and a storey height of 4,5 m.

The modulus of the subsoil was chosen very high in this case to get an upper limit. It is taken into account with 1000000 MN/m^2 to model a rigid restraint.

The results reflect the effect of the clamped bottom end, too. Reducing the modulus of the subsoil to 10000 MN/m^2 the results of the bending moments remains almost the same. Reducing it to 1000 MN/m^2 reduces the restraint moment at about 10 %. The practical range of the modulus ranges from 20 MN/m^2 up to 100 MN/m^2.

The analogous calculation with a modulus of 100 MN/m^2 results in a elastic restraint with about 40 % rigidity of a fully clamped support. Calculating with a very compressible subsoil respectively a modulus of 20 MN/m^2 we receive an elastic restraint of more than 10 % rigidity of a fully clamped support (*see Fig.14*).

Fig.15 is shows that for the variation of the modulus of the subsoil between 20 MN/m^2 and 100 MN/m^2 the ratio of the bending moment of the elastic restraint to a fully clamped support increases from about 14 % to about 44 % almost following a linear function by about 0,0375 when raising the modulus by 10 MN/m^2. For the bottom support of the column that leads to the same effect we recognized in the second paragraph for the intermediate support. Depending on the modulus of the subsoil we get a percentage of rigidity for the restraining moment and with the aid of engineering mechanics [4] [5] [6] we can calculate the zero crossing of the bending moment curve respectively the reversal point for the deflection shape. With the knowledge of the second paragraph we are now in the position to reduce the buckling length depending on the explained effect. We can point out that there is a significant influence of the restraining effect at the bottom support on the buckling length of the decisive column section of the bottom storey.

4. Summary and Future Work

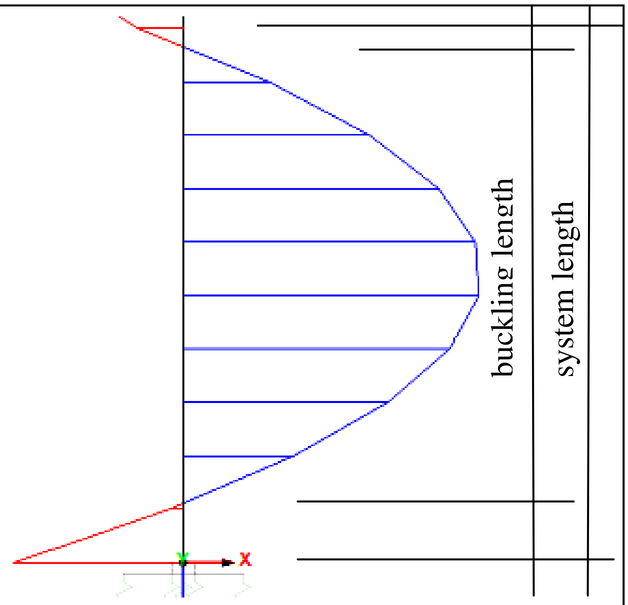

Fig.16: Example for restraining moment at bottom end and intermediate support

We can sum up that there is a restraining moment at the intermediate support of the column due to the stepped normal force within the column and a restraining moment at the bottom support of the column due to the real dimensions of the column cross section. The restraining moment at the intermediate support is primarily depending on the number of storeys. The restraining moment at the bottom support is mainly depending on the modulus of the subsoil. Both restraining moments create their own zero crossing of the bending moment curve which causes a reversal point within the deflection shape and a reduction of the buckling length. In future studies we will analyse the influence of eccentric slab joints on the buckling length of the bottom section of the multi storey column and the interaction of both restraining moments (*see Fig.16*).

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